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N=2 central charge superspace and a minimal supergravity multiplet

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Abstract

We extend the notion of central charge superspace to the case of local supersymmetry. Gauged central charge transformations are identified as diffeomorphisms at the same footing as space-time diffeomorphisms and local supersymmetry transformations. Given the general structure we then proceed to the description of a particular vector-tensor supergravity multiplet of $24 + 24$ components, identified by means of rather radical constraints.

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1. The problem of gauging central charge transformations in theories with extended supersymmetry, for the time being mostly $N = 2$, has been the subject of a number¹ of interesting investigations [1], [2], [3], [4], [5], [6], [7], [6]. In spite of these efforts, the issue of gauged central charge is far from being appropriately understood and clearly deserves further study. The work presented here is motivated by the superspace formulation of the $N = 2$ vector-tensor multiplet [8], [9] [10]. This multiplet exhibits nontrivial central charge and in [11] it has been shown that central charge transformations have an interpretation as diffeomorphisms in central charge superspace [12], [13]. Inspired by this observation we propose here an extension of $N = 2$ supergravity to local central charge superspace, employing the usual techniques of differential geometry in superspace. The frame in this kind of superspace,

$$E^{\mathcal{A}} = dz^{\mathcal{M}} E_{\mathcal{M}}^{\mathcal{A}}, \quad (1)$$

has components $E^{\mathcal{A}} \sim E^a, E_{\mathbf{A}}^{\alpha}, E_{\dot{\mathbf{A}}}^{\alpha}, E^z, E^{\bar{z}}$. Besides the components E^a , $E_{\mathbf{A}}^{\alpha}$ and $E_{\dot{\mathbf{A}}}^{\alpha}$, which contain the usual frame (or vierbein) of space-time and the Rarita-Schwinger spinors in their lowest superfield order, we add components E^z and $E^{\bar{z}}$ corresponding to the central charge sector. The lowest superfield components of E_m^z and $E_m^{\bar{z}}$ are then identified as central charge gauge fields, with local central charge transformations realised as superspace diffeomorphisms in the central charge directions. These vector fields are candidates to describe the graviphoton of $N = 2$ supergravity.

In this general setting one defines the superspace torsion 2-form

$$T^{\mathcal{A}} = dE^{\mathcal{A}} + E^{\mathcal{B}} \Phi_{\mathcal{B}}^{\mathcal{A}}, \quad (2)$$

as the covariant exterior derivative of the frame $E^{\mathcal{A}}$. The gauge connection takes its values in the Lie algebra of the structure group transformations which, besides the usual Lorentz and $SU(2) \times U(1)$ parts may now contain nontrivial phase transformations in the central charge sector with gauge connections Φ_z^z and $\Phi_{\bar{z}}^{\bar{z}}$, respectively, as well.

A combined infinitesimal diffeomorphism and structure group transformation of parameters $\xi^{\mathcal{M}}$ and $\Lambda_{\mathcal{B}}^{\mathcal{A}}$, respectively, changes the frame as

$$\delta E^{\mathcal{A}} = L_{\xi} E^{\mathcal{A}} + E^{\mathcal{B}} \Lambda_{\mathcal{B}}^{\mathcal{A}}, \quad (3)$$

or, in equivalent covariantized notation :

$$\delta E^{\mathcal{A}} = \iota_{\xi} T^{\mathcal{A}} + D\xi^{\mathcal{A}} + E^{\mathcal{B}} (\Lambda_{\mathcal{B}}^{\mathcal{A}} - \iota_{\xi} \Phi_{\mathcal{B}}^{\mathcal{A}}), \quad (4)$$

Following common usage we define *supergravity transformations* as combinations of diffeomorphisms and field dependent structure group transformations such that $\Lambda_{\mathcal{B}}^{\mathcal{A}} = \iota_{\xi} \Phi_{\mathcal{B}}^{\mathcal{A}}$, *i.e.*

$$\delta E_{\mathcal{M}}^{\mathcal{A}} = \mathcal{D}_{\mathcal{M}} \xi^{\mathcal{A}} + E_{\mathcal{M}}^{\mathcal{B}} \xi^{\mathcal{C}} T_{\mathcal{CB}}^{\mathcal{A}}. \quad (5)$$

Here, the parameters $\xi_{\mathbf{A}}^{\alpha}, \xi_{\dot{\mathbf{A}}}^{\alpha}$ pertain to supersymmetry transformations whereas $\xi^z, \xi^{\bar{z}}$ will parametrize gauged central charge transformations.

¹It is not intended here to give an exhaustive list of references and we apologize for undue omissions.

2. Besides the physical fields of a supergravity multiplet, in the present case vierbein, Rarita-Schwinger fields and graviphoton, an off-shell multiplet will also exhibit auxiliary fields. Different auxiliary field structures of different supergravity multiplets will reflect themselves in different choices of torsion constraints. The general constraints to be described in this section should still allow for the implementation of different inequivalent multiplets after suitable supplementary restrictions. The general constraints consist of two parts. In a first step we require that the nonvanishing torsion components at (engineering) dimension zero take the form

$$T_{\gamma\beta}^{\text{CB}z} = \epsilon_{\gamma\beta} \epsilon^{\text{CB}} c^z, \quad T_{\gamma\text{B}}^{\text{C}\dot{\beta}a} = -2i \delta_{\text{B}}^{\text{C}} (\sigma^a \epsilon)_{\gamma}^{\dot{\beta}}, \quad T_{\text{CB}}^{\dot{\gamma}\dot{\beta}\bar{z}} = \epsilon^{\dot{\gamma}\dot{\beta}} \epsilon_{\text{CB}} \bar{c}^{\bar{z}}, \quad (6)$$

where we allow for c^z and $\bar{c}^{\bar{z}}$ the possibility to be superfields², contrary to the flat case where they were supposed to be constants. These constraints provide rather mild restrictions and we will not pursue here their consequences on the structure of the other torsion components.

We will instead, in a second step, proceed to implement reality conditions, motivated by the reality conditions encountered in the geometrical formulation of the vector-tensor multiplet, but generalized to local central charge superspace. Expressed in terms of torsion components these reality conditions read

$$c^z T_{z\mathcal{B}} \underline{\mathcal{A}} = \bar{c}^{\bar{z}} T_{\bar{z}\mathcal{B}} \underline{\mathcal{A}}, \quad (7)$$

where underlined calligraphic indices like $\underline{\mathcal{A}}$ run over ordinary superspace only, *i.e.* $\underline{\mathcal{A}} \sim a, \alpha, \underline{\mathcal{A}}, \underline{\alpha}$, and

$$\mathcal{D}_{\mathcal{B}} c^z = \bar{c}^{\bar{z}} T_{\bar{z}\mathcal{B}}^z - c^z T_{z\mathcal{B}}^{\bar{z}}, \quad (8)$$

$$\mathcal{D}_{\mathcal{B}} \bar{c}^{\bar{z}} = c^z T_{z\mathcal{B}}^{\bar{z}} - \bar{c}^{\bar{z}} T_{\bar{z}\mathcal{B}}^z, \quad (9)$$

in the central charge directions. Clearly, these reality constraints are more stringent than the dimension zero constraints of (6). For instance, taking $\mathcal{B} \sim z$ in (7) yields immediately that the torsion components $T_{zz} \underline{\mathcal{A}}$ must vanish. A more detailed analysis of the properties of torsions and curvatures subject to these general constraints will be discussed elsewhere.

Another possible type of further reduction could be to ask c^z and $\bar{c}^{\bar{z}}$ to be covariantly constant, *i.e.*

$$Dc^z = dc^z + c^z(\Phi_z^z - 2\Omega) = 0, \quad D\bar{c}^{\bar{z}} = d\bar{c}^{\bar{z}} + \bar{c}^{\bar{z}}(\Phi_{\bar{z}}^{\bar{z}} + 2\Omega) = 0. \quad (10)$$

Here Ω is the $U(1)$ gauge potential, with fieldstrength $\Gamma = d\Omega$, identified in

$$\Phi_{\beta\text{A}}^{\text{B}\alpha} = \delta_{\text{A}}^{\text{B}} \Phi_{\beta}^{\alpha} + \delta_{\beta}^{\alpha} \Phi_{\text{A}}^{\text{B}} + \delta_{\text{A}}^{\text{B}} \delta_{\beta}^{\alpha} \Omega, \quad (11)$$

with Φ_{β}^{α} and $\Phi_{\text{A}}^{\text{B}}$ the Lorentz and $SU(2)$ gauge potentials, respectively. In addition, inspired by the vector-tensor multiplet central charge transformations one might adopt a parametrization such that

$$E^z = Vc^z, \quad E^{\bar{z}} = \bar{V}\bar{c}^{\bar{z}}, \quad (12)$$

for the frame and $\xi^z = \omega c^z$ and $\xi^{\bar{z}} = \bar{\omega} \bar{c}^{\bar{z}}$ for the parameters. In this case V and \bar{V} inherit the (opposites of the) chiral $U(1)$ weights of c^z and $\bar{c}^{\bar{z}}$ and one finds

$$T^z = Fc^z, \quad \text{with} \quad F = dV + 2V\Omega. \quad (13)$$

²Using conventions for chiral $U(1)$ weights such that $w(E_{\text{A}}^{\alpha}) = +1$, $w(E_{\alpha}^{\text{A}}) = -1$, the decompositions employed in (6) imply $w(c^z) = -2$, $w(\bar{c}^{\bar{z}}) = +2$.

The gauge transformations of V are derived from those of E^z in combination with the condition $Dc^z = 0$. If λ denotes an infinitesimal $U(1)$ transformation, then

$$\delta\Omega = -d\lambda, \quad \delta V = d\omega + 2\omega\Omega + 2\lambda V, \quad (14)$$

and the transformation law for F reads

$$\delta F = 2\omega\Gamma + 2\lambda F. \quad (15)$$

Similar considerations apply to $E^{\bar{z}}$ and $T^{\bar{z}} = \bar{F}\bar{c}^{\bar{z}}$. Instead of pursuing a more detailed discussion of these general superspace structures we shall present, as an example of central charge superspace at work, the superspace description of a rather restricted supergravity multiplet.

3. The vector-tensor supergravity multiplet is extracted from general central charge superspace by means of quite *radical constraints*. First of all we restrict the structure group to be the product of Lorentz and $SU(2)$ transformations only (no $U(1)$ and no structure group transformations in the central charge sector). As to the torsion components we will describe explicitly a definit parametrisation in terms of a few superfields, without going into the details as to what are independent constraints and what are consequences thereof. To begin with, at dimension zero, the nonvanishing components are those of (6), with c^z and $\bar{c}^{\bar{z}}$ taken to be constants. We then parametrise $E^z = Vc^z$ and $E^{\bar{z}} = \bar{V}\bar{c}^{\bar{z}}$. Defining $F = dV$ and $\bar{F} = d\bar{V}$ we obtain $T^z = Fc^z$ and $T^{\bar{z}} = \bar{F}\bar{c}^{\bar{z}}$. The only nonvanishing components, besides F_{ba} and \bar{F}_{ba} , of the superspace 2-forms F and \bar{F} are

$$F_{\beta\alpha}^{\text{BA}} = \epsilon_{\beta\alpha}\epsilon^{\text{BA}}, \quad \bar{F}_{\text{BA}}^{\dot{\beta}\dot{\alpha}} = \epsilon^{\dot{\beta}\dot{\alpha}}\epsilon_{\text{BA}}. \quad (16)$$

At dimension $\frac{1}{2}$ all torsion components vanish and at dimension one we are left with.

$$T_{\gamma b \dot{\alpha}}^{\text{C} \text{A}} = -2i\epsilon^{\text{CA}}\sigma_{\gamma\dot{\alpha}}^c \bar{F}_{cb}, \quad T_{\text{C} b \text{A}}^{\dot{\gamma} \alpha} = -2i\epsilon_{\text{CA}}\bar{\sigma}^{c\dot{\gamma}\alpha} F_{cb}, \quad (17)$$

as well as

$$T_{\gamma b \text{A}}^{\text{C} \alpha} = 2\delta_{\text{A}}^{\text{C}} U^c (\sigma_{cb})_{\gamma}^{\alpha}, \quad T_{\text{C} b \dot{\alpha}}^{\dot{\gamma} \text{A}} = -2\delta_{\text{C}}^{\text{A}} U^c (\bar{\sigma}_{cb})^{\dot{\gamma}}_{\dot{\alpha}}. \quad (18)$$

This identifies the basic superfields F_{ba} , \bar{F}_{ba} and U_a which completely describe the components of torsion and curvature, as for instance the Lorentz curvatures at dimension one,

$$R_{\delta\gamma ba}^{\text{DC}} = 8\epsilon_{\delta\gamma}\epsilon^{\text{DC}} \bar{F}_{ba}, \quad R_{\delta\text{C} ba}^{\text{D}\dot{\gamma}} = 4\delta_{\text{C}}^{\text{D}} (\sigma^d\epsilon)_{\delta}^{\dot{\gamma}} U^c \epsilon_{dcba}, \quad R_{\text{DC} ba}^{\delta\dot{\gamma}} = 8\epsilon^{\delta\dot{\gamma}} \epsilon_{\text{DC}} F_{ba}, \quad (19)$$

and the Rarita-Schwinger torsions at dimension $\frac{3}{2}$,

$$T_{cb \text{A}}^{\alpha} = -\mathcal{D}_{\text{A}}^{\alpha} F_{cb}, \quad T_{cb \dot{\alpha}}^{\text{A}} = -\mathcal{D}_{\dot{\alpha}}^{\text{A}} \bar{F}_{cb}. \quad (20)$$

Moreover we have the chirality conditions

$$\mathcal{D}_{\text{A}}^{\alpha} \bar{F}_{cb} = 0, \quad \mathcal{D}_{\dot{\alpha}}^{\text{A}} F_{cb} = 0, \quad (21)$$

and the relations

$$\mathcal{D}_{\alpha}^{\text{A}} U^d = \frac{1}{4}\epsilon^{dcba}\sigma_{c\alpha\dot{\alpha}}\mathcal{D}^{\dot{\alpha}\text{A}}\bar{F}_{ba}, \quad \mathcal{D}_{\text{A}}^{\dot{\alpha}} U^d = \frac{1}{4}\epsilon^{dcba}\bar{\sigma}_c^{\dot{\alpha}\alpha}\mathcal{D}_{\alpha\text{A}}F_{ba}. \quad (22)$$

As a consequence of the last two equations one obtains

$$\mathcal{D}^a U_a = i\varepsilon^{dcba} \bar{F}_{dc} F_{ba}. \quad (23)$$

This covariant superspace identity suggests to interpret U^a as the curl of a two form gauge potential, contributing an antisymmetric tensor to the multiplet. Its explicit identification requires some more (quite intriguing) technicalities.

It is however quite obvious to identify all the other component fields in the superspace geometry presented so far. The vierbein and Rarita-Schwinger fields are defined as usual, *i.e.*

$$E^a| = dx^m e_m{}^a(x), \quad E_A^\alpha| = \frac{1}{2} dx^m \psi_{m\dot{A}}^\alpha(x), \quad E_{\dot{\alpha}}^A| = \frac{1}{2} dx^m \bar{\psi}_{m\dot{\alpha}}^A(x). \quad (24)$$

Likewise, the central charge and $SU(2)$ gauge potentials are identified in

$$\Phi_B{}^A| = dx^m \varphi_{mB}{}^A(x), \quad V| = dx^m v_m(x), \quad \bar{V}| = dx^m \bar{v}_m(x). \quad (25)$$

Given these identifications, supersymmetry (or better supergravity transformations) are obtained using textbook methods. The only missing piece in this construction is the antisymmetric tensor.

4. The identity (23) has been obtained in the framework of superspace geometry pertaining to the gravity sector. As it stands it suggests an interpretation as Bianchi identity for an antisymmetric tensor in the presence of suitable Chern-Simons forms. In order to obtain a fully supercovariant description one should embed (23) in the superspace geometry of a 2-form gauge potential. In what follows we shall derive (23) from a suitably defined 2-form geometry.

To begin with we consider the superspace 2-form gauge potential B with invariant field-strength $H = dB$ and Bianchi identity $dH = 0$, more explicitly

$$E^A E^B E^C E^D \left(4\mathcal{D}_D H_{CBA} + 6T_{DC}{}^F H_{FBA} \right) = 0. \quad (26)$$

Imposing constraints such that all the components of H_{CBA} at dimension $-\frac{1}{2}$ (all indices spinorial) vanish and at dimension 0 the nonvanishing components

$$H_{\gamma B a}^{C\dot{\beta}} = -2i \delta_B^C (\sigma_a \epsilon)_\gamma{}^{\dot{\beta}}, \quad c^z H_{zBA}^{\dot{\beta}\dot{\alpha}} = -8 \epsilon^{\dot{\beta}\dot{\alpha}} \epsilon_{BA}, \quad \bar{c}^{\bar{z}} H_{\bar{z}\beta\alpha}^{BA} = -8 \epsilon_{\beta\alpha} \epsilon^{BA}, \quad (27)$$

are all constant, leads, upon repeated use of the Bianchi identities (26), to the identifications

$$U^d = \frac{i}{24} \varepsilon^{dcba} H_{cba}, \quad (28)$$

and

$$\frac{1}{2} E^A E^B c^z H_{zBA} = -8\bar{F}, \quad \frac{1}{2} E^A E^B \bar{c}^{\bar{z}} H_{\bar{z}BA} = -8F, \quad (29)$$

with \bar{F} and F as identified above in (17). In fact, the components of H appearing in the last three equations are the only nonvanishing ones. Having made these identifications, the purely vectorial part of (26),

$$\varepsilon^{dcba} \left(4\mathcal{D}_d H_{cba} + 6T_{dc}{}^z H_{zba} + 6T_{dc}{}^{\bar{z}} H_{\bar{z}ba} \right) = 0, \quad (30)$$

reproduces exactly (23). Observe that the topological term $\varepsilon^{dcba}\bar{F}_{dc}F_{ba}$ in this equation arises automatically. The mechanism presented here to merge the geometries of the 2-form gauge potential and of supergravity in superspace is similar to the one used in the geometrical description of the $N = 1$ new-minimal multiplet. It is in this sense that the multiplet presented here may be considered as the analogue of new-minimal $N = 1$ supergravity. The missing piece in completing the multiplet (24), (25), *i.e.* the antisymmetric tensor gauge field, is now identified in

$$B| = \frac{1}{2}dx^m dx^n b_{nm}(x), \quad (31)$$

5. Having established the complete superspace geometry relevant for our multiplet we can now write down the supergravity transformations

$$\delta_\xi E^A = D\xi^A + \iota_\xi T^A, \quad \delta_\xi \Phi_B^A = \iota_\xi R_B^A, \quad \delta_\xi B = \iota_\xi H, \quad (32)$$

defined in the usual way as suitable combinations of superspace diffeomorphisms and field dependent structure group and 1-form gauge transformations. Suitable projections of these equations allow to derive supersymmetry as well as local central charge transformations of the component fields. The supersymmetry transformations are given as (with usual conventions for summation over spinor indices)

$$\delta e_m^a = -i\psi_m \sigma^a \bar{\zeta} - i\bar{\psi}_m \bar{\sigma}^a \zeta, \quad (33)$$

$$\delta \psi_m^\alpha = 2\mathcal{D}_m \zeta_\alpha - \frac{i}{6} \zeta_\alpha^\gamma (\sigma_{md})_\gamma^\alpha \varepsilon^{dcba} H_{cba} + 4ie_m^a \bar{\zeta}_{\dot{\alpha}A} \bar{\sigma}^{b\dot{\alpha}\alpha} F_{ba}, \quad (34)$$

$$\delta \bar{\psi}_m^{\dot{\alpha}} = 2\mathcal{D}_m \bar{\zeta}_{\dot{\alpha}} + \frac{i}{6} \bar{\zeta}_{\dot{\alpha}}^\gamma (\bar{\sigma}_{md})_{\dot{\gamma}}^{\dot{\alpha}} \varepsilon^{dcba} H_{cba} + 4ie_m^a \zeta^{\alpha A} \sigma_{\alpha\dot{\alpha}}^b \bar{F}_{ba}, \quad (35)$$

$$\delta b_{mn} = i\psi_n \sigma_m \bar{\zeta} + i\bar{\psi}_n \bar{\sigma}_m \zeta + 4\bar{v}_m \psi_n \zeta + 4v_m \bar{\psi}_n \bar{\zeta} - m \leftrightarrow n, \quad (36)$$

$$\delta v_m = \frac{1}{2}\psi_m \zeta, \quad \delta \bar{v}_m = \frac{1}{2}\bar{\psi}_m \bar{\zeta}, \quad (37)$$

$$\begin{aligned} \delta \phi_{mB}^A &= \left(\delta_D^A \delta_B^C - \frac{1}{2} \delta_B^A \delta_D^C \right) \left\{ 2ie_m^a \left(\zeta_C \sigma^b \bar{T}_{ba}{}^D - \bar{\zeta}_C \bar{\sigma}^b T_{ba}{}^D \right) \right. \\ &\quad \left. + 4\zeta_C \sigma^{ba} \psi_m{}^D \bar{F}_{ba} - 4\bar{\zeta}_C \bar{\sigma}^{ba} \bar{\psi}_m{}^D F_{ba} \right. \\ &\quad \left. + \frac{1}{6} (\zeta_C \sigma_d \bar{\psi}_m{}^D - \bar{\zeta}_C \bar{\sigma}_d \psi_m{}^D) \varepsilon^{dcba} H_{cba} \right\}. \end{aligned} \quad (38)$$

The supercovariant fieldstrengths appearing in the gravitino transformation law are identified as usual in the lowest components of the corresponding superspace tensors, their explicit form being

$$\bar{F}_{ba}| = e_b^n e_a^m \left(\partial_n \bar{v}_m - \partial_m \bar{v}_n + \frac{1}{4} \bar{\psi}_n^{\dot{\alpha}A} \bar{\psi}_m{}_{\dot{\alpha}}^{\dot{\alpha}} \right), \quad (39)$$

$$F_{ba}| = e_b^n e_a^m \left(\partial_n v_m - \partial_m v_n + \frac{1}{4} \psi_n^\alpha \psi_m{}_{\alpha} \right), \quad (40)$$

and

$$\varepsilon^{dcba} H_{cba}| = 3\varepsilon^{dmlk} (\partial_m b_{lk} + 16v_m \partial_l \bar{v}_k + 16\bar{v}_m \partial_l v_k + i\psi_m \sigma_l \bar{\psi}_k). \quad (41)$$

Note the presence of the mixed Chern-Simons form in the fieldstrength of the antisymmetric tensor. Finally, the covariant fieldstrength of the Rarita-Schwinger field is given as

$$T_{cbA}^\alpha| = \frac{1}{2} e_c^n e_b^m \partial_n \psi_m^\alpha - ie_c^n \bar{\psi}_n^{\dot{\alpha}A} \bar{\sigma}^{f\dot{\alpha}\alpha} F_{fb}| + e_c^n \psi_n^\beta (\sigma_{be})_\beta^\alpha U^e| - c \leftrightarrow b, \quad (42)$$

and similar for the complex conjugate fieldstrength.

Although we are working in central charge superspace, nontrivial central charge transformations are restricted to the sector of V , \bar{V} and B . Parametrizing $\xi^z = \omega c^z$ and $\xi^{\bar{z}} = \bar{\omega} \bar{c}^{\bar{z}}$ one finds

$$\delta_{c.c.} V = d\omega, \quad (43)$$

$$\delta_{c.c.} \bar{V} = d\bar{\omega}, \quad (44)$$

$$\delta_{c.c.} B = -8\bar{\omega}F - 8\bar{F}\omega. \quad (45)$$

The other components of the multiplet are inert under central charge transformations due to the drastical torsion constraints we have imposed.

6. One might introduce the combinations $V^\pm = V \pm \bar{V}$, as well as $F^\pm = F \pm \bar{F}$ and interpret V^+ as the graviphoton. Dynamical equations, *i.e.* Einstein and Rarita-Schwinger equations as well as Maxwell's second set of equations for the graviphoton are then obtained from imposing suitable additional constraints. It is not difficult to convince oneself that that this is achieved through the superfield equations

$$U_a = 0, \quad F_{ba}^\pm = \frac{i}{2} F^\mp{}^{dc} \varepsilon_{dcba}. \quad (46)$$

This trivializes the antisymmetric tensor in the sense that it becomes a pure gauge. Consistency of this mechanism with the presence of the mixed Chern-Simons forms in its fieldstrength is ensured by the selfduality conditions. The $SU(2)$ gauge field becomes pure gauge as well, inducing a covariant gauge with respect to $SU(2)$ for the gravitini fields.

7. We have presented the general structure of $N = 2$ central charge superspace and emphasized a soldering mechanism involving superspace geometries relevant for supergravity on the one hand and 2-form superspace geometry, suitable for the vector-tensor multiplet, on the other hand. Gauged central charge transformations have an interpretation as a subset of superspace diffeomorphisms. Correspondingly, the central charge gauge fields appear in the local frame of superspace. Moreover, this geometric construction implies the presence of Chern-Simons forms pertaining to the central charge gauge fields in the fieldstrength of the antisymmetric tensor.

The minimal vector-tensor multiplet we have described is a particular special case of in this geometrical setting. In some sense it may be viewed as an analogue of the new-minimal multiplet in $N = 1$ superspace geometry. Contrary to the $N = 1$ case, the emergence of selfduality conditions in the central charge gauge sector seems to prevent a lagrangean formulation in the present case (see also [14]). On the other hand, this same property may give rise to relations with integrable hierarchies, an aspect which deserves further study.

The general geometric setting outlined in the beginning of this paper ("natural constraints" in central charge superspace) should allow, upon suitable reductions via different types of torsion constraints, to identify other multiplets, similar to what has been done in $N = 1$ supergravity and in $N = 2$ superspace without central charges. Investigations on these topics are under way.

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